The Ballistic Pendulum

Crime Lab Physics

You are a criminologist, and have been given a gun which has been used in the commission of a crime. Both prosecution and defense claim that "muzzle velocity" (speed of the bullet as it leaves the gun) is crucial to the case. Your job...FIND IT!

Shown below is a diagram representing a "ballistic pendulum." The bullet is fired into the block, the bullet-block combination rises by a height $h$ as shown, a little physics is applied, and ...voila!...there you have it!

Let us proceed: (correct answer choices are given on the next page)

1. The mass of the bullet is 0.01 kg and the mass of the block is 0.99 kg. The bullet is fired into the block and becomes imbedded in it. The bullet-block combination rises by 0.05 meters. What potential energy is stored in the pendulum (bullet-block combination));

$$PE = mgh =$$

a. 50 joules          b. 5 joules          c. 0.5 joules          d. 0.05 joules

2. What was the kinetic energy of the pendulum (bullet-block combination) "immediately" after the bullet became imbedded (i.e. what was the MAXIMUM K.E. of the combination)?

$$KE = PE =$$ (conservation of energy -- no calculation required!)

a. 50 joules          b. 5 joules          c. 0.5 joules          d. 0.05 joules

3. What was the velocity of the pendulum (bullet-block combination) "immediately" after the bullet became imbedded (i.e. when its K.E. was maximum)?

$$KE = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2KE}{m}}$$

a. 0.5 m/sec          b. 1.0 m/sec           c. 1.5 m/sec           d. 2.0 m/s

4. What was the momentum of the pendulum (bullet-block combination) "immediately" after the bullet became imbedded (i.e. when its K.E. was maximum)?

$$p = mv =$$

a. 0 kg m/sec          b. 0.25 kg m/sec       c. 0.5 kg m/sec       d. 1.0 kg m/sec
5. What was the momentum of the block before the bullet hit it?

\[ p_{\text{block}} = mv = \text{(think about this -- mental calculation -- no calculator required!)} = \]

a. 0 kg m/sec  
   b. 0.25 kg m/sec  
   c. 0.5 kg m/sec  
   d. 1.0 kg m/sec

6. What was the momentum of the bullet before it hit the block?

\[ p_{\text{bullet}} = \text{(conservation of momentum -- no calculation required!)} = \]

a. 0 kg m/sec  
   b. 0.25 kg m/sec  
   c. 0.5 kg m/sec  
   d. 1.0 kg m/sec

7. What was the velocity of the bullet before it hit the block (i.e. the "muzzle velocity")?

\[ p_{\text{bullet}} = mv \]

\[ v = \frac{p_{\text{bullet}}}{m} \]

a. 25 m/sec  
   b. 50 m/sec  
   c. 75 m/sec  
   d. 100 m/sec  
   e. 250 m/sec

### LAB OR DEMONSTRATION SET-UP & PROCEDURE

**Materials**
- toy dart gun with suction cup darts
- balance
- 1 PVC 1/2 in. pipe, 12 in. long
- 5 PVC 1/2 in. pipe, 4 in. long
- 2 PVC 1/2 in. pipe, 2 in. long
- 3 PVC 1/2 in. 90 degree elbows
- 2 PVC 1/2 in. T's
- plastic water bottle, 500 mL, with top
- plastic grocery bag
- string
- tape, translucent and/or masking
- jumbo paper clip
- 5x8 index card
- washer, steel, 3/4 in (approximate mass 50 g)
- marker pen
- ruler
- scissors and/or utility knife
- calculator

**Assembly**

1. Assemble the PVC pieces into the stand shown in Figure 2.

2. Use the utility knife and/or scissors to cut the top end off the water bottle. Make the cut just below where the curvature stops and the straight sides begin. See Figures 3 and 4.

3. Tape the washer to the middle of the bottle. See Figure 3 and 4.

4. Stuff the plastic shopping bag in the bottle in such a way that the bag is pressed out toward the walls of the bottle, forming at least a semblance of an open core down the middle (this isn't too crucial, but you should avoid compressing the entire bag into the bottom of the bottle).
5. Use two pieces of string to suspend the bottle, with the washer on the bottom, as shown in Figures 3 and 4. Tape the strings in place on the bottle and on the stand. Make sure the strings are near the ends of the bottle, and that they are as close to parallel as you can make them.

6. Tape the paper clip to the bottle as shown in Figures 3 and 4.

7. Fold the 4x6 card in half so that it will stand as shown in Figures 3 and 4.

**To Do and Notice**

1. Set up the apparatus as shown in Figure 3, with the stand taped to the table surface, and with the folded edge of the card placed against the end of the bottle. Mark the card where the end of the paper clip hits it, as shown in Figure 4.

2. The next step involves finding how high the bottle has risen at the end of its maximum swing after being hit by the dart. After marking the card as described in Step 1 and shown in Figure 4, remove the card, hold the gun a few inches away from the bottle, and fire the dart into the bottle. Notice how far the bottle moves, and place the fold in the card where you think the bottle will just barely hit it. Fire the dart again, and move the card as necessary so that the bottle just barely touches it at the end of its swing (if it gets kicked hard, move it farther away...if the bottle doesn't reach it, move it closer...). Repeat several times (always holding the gun the same distance from the bottle when you fire the dart) until you have a good idea of the location of the end of the bottle at the end of it's maximum swing, and the fold in the card is located at this position.
3. Without moving the card, gently push the bottle along its swing path until the end of the bottle is against the card at the card's new location. Mark the new position of the end of the paper clip on the card. The vertical difference between the two positions is the height which the bottle rose when hit by the dart.

4. Measure the vertical distance between the two marks on the card to find the distance, \( h \), that the bottle rose (see Figures 1 and 5). Record.

5. Find the mass of the dart and record.

6. Find the mass of the bottle assembly (without the dart). (You don't need to disassemble the whole setup -- you can just rest the bottle assembly on the balance with the strings slack). Record the mass.

7. Use your three data items to carry out the same series of calculations done in steps 1-7 of the introductory example. The final result will give you the muzzle velocity of the dart.

8. OPTIONAL CHECK: Go where there is plenty of vertical room (outside, in a gym, etc.). Fire the dart straight up in the air and estimate how far above the muzzle of the gun it rises (don't forget to take into account how far above the ground you're holding the gun when you fire it). Use the relationship below to determine the muzzle velocity (this is based on conservation of energy, and assumes that air resistance is negligible):

\[
\text{PE}_{\text{gain}} = \text{KE}_{\text{loss}} \\
 \text{mgh} = \frac{1}{2}mv^2 \\
 v^2 = 2gh \\
 v = \sqrt{2gh}
\]

**NOTE:** Try to be as careful as you can, but remember that this is "close enough for government work" science, using cheap, homemade apparatus, and that you can't make steak out of hamburger, etc. You might get some interesting results whose accuracy and precision may actually be pretty good...or maybe not. But in any case, you can get a very good idea of the process, and the physics involved, for very little cost! The design of the setup here has economy and simplicity as major considerations. There are aspects that could be redesigned to make the apparatus more robust and to improve the precision and accuracy, and you are encouraged to make any changes that make it more suitable to your own use. You should also be aware that there are commercial versions of the ballistic pendulum designed for educational use, if your needs and budget make this of interest (e.g., see PASCO Ballistic Pendulum -- this is high quality equipment which has versions that range in price from a couple of hundred to several hundred dollars).