

## Solidly Platonic

Follow in the steps of Pythagoras, Plato and Euler and explore some special shapes

### Materials

Card stock or other heavy paper  
Scissors  
Tape  
Shape templates from this activity

Copy the shape templates onto the cardstock or other stiff paper.

### To do and notice

Cut out several pieces of each shape.

The corner of a cube is where 3 squares meet at a single point. Make different “corners” out of the other shapes by taping together 3 or more pieces of the *same shape*. You must use identical shapes in any given corner, and all the sides have to meet at a single point. Something qualifies as a corner if it can hold water – flat pieces don’t count!

How many can you make?

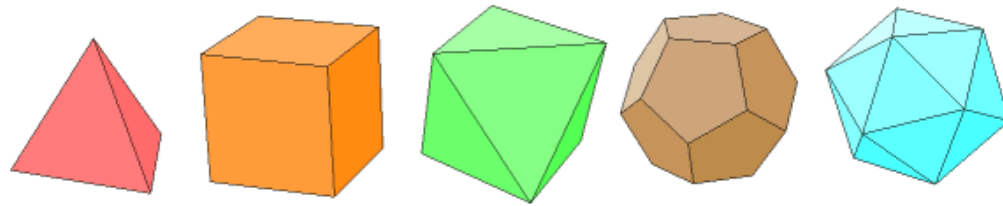
The mathematical name for the corner where the sides meet is a **vertex**. Each side is called a **face**. The line where two faces meet is called an **edge**.

The “corners” that you made should have some open edges. Use additional pieces of the same shape to extend your corners, but keep the number of faces that meet at each vertex the same. You should be able to add more faces until you have a closed container, or **polyhedron**. Each face, edge, and vertex on your polyhedron should be identical. Count the number of faces, edges, and vertices in each of your structures. Do you see a pattern?

### What’s going on?

The shapes used in this activity are all regular polygons, meaning all the sides in each polygon are the same length. The **Platonic solids** are polyhedrons whose faces are identical regular polygons that meet at identical vertices. Because of these restrictions, there are only 5 that exist. You can prove this mathematically by summing the interior angles of all the faces that meet at each vertex. The sum of these angles must be less than  $360^\circ$ . If the sum is greater, then faces will not be able to come together at a point without overlapping. If the sum is exactly  $360^\circ$ , such as with four squares, the piece will be flat and will not form a vertex. Since the interior angle of a regular hexagon is  $120^\circ$ , and there must be at least 3 faces that meet at the vertex, it is impossible for a regular polygon with 6 or more sides to be the face of a Platonic solid. The only possible faces are triangles, squares, and pentagons. With these shapes, you should only have

been able to make 5 unique corners. These represent the 5 different vertices that are present in the Platonic solids. Here is some more information about the solids:



Name	Tetrahedron	Cube	Octahedron	Dodecahedron	Icosohedron
Faces/vertex	3 triangles	3 squares	4 triangles	3 pentagons	5 triangles
Faces	4	6	8	12	20
Vertices	4	8	6	20	12
Edges	6	12	12	30	30

Euler noticed that there is a relationship between the number of faces, edges, and vertices in each solid. This relationship is represented in Euler's formula:  $F+V=E+2$

