Scaling Using Powers of 10

This is based on an activity developed by Ellen Koivisto, who teaches at School of the Arts in San Francisco.

Background:

Big Numbers

The diameter of the earth is about 13,000 km. The distance from the earth to the moon is about 390,000 km, and from the earth to the sun about 150 million km. The distance to the nearest star is roughly 40,000,000,000,000 km or 40 million million km! This distance is a small fraction of the distance across our galaxy, the Milky Way galaxy. The distance across the Milky Way galaxy is only a tiny fraction of the distance across the universe.

Small Numbers

The height of a human is almost 2 m. The diameter of fine human hair is about 20 micrometers, about 100,000 times smaller than the height of the human. An average human cell diameter is about 10 micrometers. Wavelengths of visible light range from about 400-700 nm, more than 10 million times smaller than the height of the human.

How can we relate to the sizes of very large and very small objects? When we try to compare one set of objects to another -- whether they're astronomical, biological, or other -- the *scale* changes by many powers of ten. The changes in the sizes and distances are exponential, or logarithmic.

We are going to make rulers that will allow us to compare the relative sizes of very large and very small objects.

Materials

Two 40 cm long strips of paper, about 5 cm wide, printed with cm marks, if possible
Metric ruler
List of large and small objects

To Do and Notice:

1. As an introduction, select 5-6 objects for the students to put in size order.
2. Turn the strips of paper into centimeter rulers by marking each centimeter along the length of the strip. Be sure to mark both strips - putting the marks on the left edge of one strip and along the right edge of the other strip. In other words, mark the two sheets so they are mirror images of one another.
3. Start at the top of each strip, and label the marks closest to the top 10^{21} m, which represents the largest object on our ruler.
4. Working downwards, label the next mark 10^{20} m, and the one below that 10^{19} m.

5. Keep labeling the marks one power of 10 lower until you reach 10^0 m.
6. Label the remaining marks within negative exponents starting with 10^{-1} m.
7. Choose the least 10 objects from the following list to add at the appropriate distance on the exponential rulers you have just created. More ideas for measurements can be found at at:

http://en.wikipedia.org/wiki/Orders_of_magnitude_(length)
It is easiest to choose numbers whose coefficients (in scientific notation form) are equal to one. If the coefficient is a number different than one, then we have to plot the logarithm of the coefficient in order to place the value correctly between the two powers of 10 on our scale.

<u>**Object**</u>	<u>**Size**</u>	<u>**Meters**</u>	<u>**Scientific Notation**</u>
Atomic nucleus	1 femto-m	0.000000000000001 m	$1.0\text{x } 10^{-15}$ m
Atom	100 pico-m	0.0000000001 m	$1.0\text{x } 10^{-10}$ m
Length of the sucrose molecule	1 nm	0.000000001 m	$1.0\text{x } 10^{-9}$ m
Approx. width of hemoglobin molecule	10 nm	0.00000001 m	$1.0\text{x } 10^{-8}$ m
HIV virus	100 nm	0.0000001 m	$1.0\text{x } 10^{-7}$ m
Bacteria	1 μm	0.000001 m	$1.0 \text{ x } 10^{-6}$ m
Fog droplet	10 μm	0.00001 m	$1.0 \text{ x } 10^{-5}$ m
Cell diameter	10 μm	0.00001 m	$1.0 \text{ x } 10^{-5}$ m
Dust particle	100 μm	0.0001 m	$1.0 \text{ x } 10^{-4}$ m
Diameter of a pinhead	1 mm	0.001 m	$1.0 \text{ x } 10^{-3}$ m
Width of little finger	1 cm	0.01 m	$1.0 \text{ x } 10^{-2}$ m
Length of toad	10 cm	0.1 m	$1.0 \text{ x } 10^{-1}$ m
Meter stick	1 m	1.0 m	$1.0 \text{ X } 10^{0}$ m
Length of giant squid	10 m	10 m	$1.0 \text{ X } 10^{1}$ m
Soccer field length	100 m	100 m	$1.0 \text{ X } 10^{2}$ m
Height of tallest tree	100 m	100 m	$1.0 \text{ X } 10^{2}$ m
Length of George Washington Bridge, NYC	1 km	1000 m	$1.0 \text{ X } 10^{3}$ m
Height of Mount Everest	10 km	10,000 m	$1.0 \text{ X } 10^{4}$ m
Diameter of Island of Hawaii	100 km	100,000 m	$1.0 \text{ X } 10^{5}$ m
Diameter of large hurricane	1000 km	1,000,000 m	$1.0 \text{ X } 10^{6}$ m
Earth diameter (approx)	10,000 km	10,000,000 m	$1.0 \text{ X } 10^{7}$ m
Saturn diameter (appprox)	100,000 km	100,000,000 m	$1.0 \text{ x } 10^{8}$ m
Sun diameter (approx)	1,000,000 km	1,000,000,000 m	$1.0 \text{ x } 10^{9}$ m
Diameter of black hole at center of our galaxy	100,000 km	100,000,000 m	$1.0 \text{ x } 10^{10}$ m
Venus to the Sun	100 Giga-m	100,000,000,000 m	$1.0 \text{ X } 10^{11}$ m
1 light hour	1 Tera-m	1,000,000,000,000 m	$1.0 \text{ X } 10^{12}$ m
Solar system diameter	10 Tera-m	10,000,000,000,000 m	$1.0 \text{ X } 10^{13}$ m
1 light year	10 Peta-m	10,000,000,000,000,000 m	$1.0 \text{ X } 10^{16}$ m
Diameter of Milky Way	100,000 light year 100 Zeta-m	1,000,000,000,000,000,000,000 m	$1.0 \text{ X } 10^{21}$ m

8. Now we can do some scaling. Place your two mirror image rulers side-by-side, so the powers of 10 measurements line up in the middle.
9. Suppose you wanted to know how large the Milky Way would be if the Earth diameter were the size of a pinhead. Slide the right-hand ruler so that the size of a pinhead on the right-hand ruler lines up with the Earth's diameter on the left-hand ruler.
10. We'll make our comparisons from the left side ruler to the right side ruler, using statements such as: "If the Earth's diameter were the size of a pinhead…"
11. Now find the diameter of the Milky Way galaxy on the left-hand ruler. Notice that it lines up with the distance from Venus to the sun. We can now complete our statement: "If the Earth's diameter were the size of a pinhead, then the diameter of the Milky Way galaxy would be equal to the distance from Venus to the sun."
12. With the rulers in this position, what other statements can you make?
13. Try other scale combinations, make statements using the following pattern:

If the (object from left-hand ruler) were the size of the (object from right-hand ruler that lines up the first object), then the (choose a second object from the left-hand ruler) would be the size of the (object with which the 2nd object lines up on the right-hand ruler).

High School Version: Coefficients vary.

When the coefficient of the number is not equal to one, where to place that value on our scale is not obvious. Since each centimeter mark on our ruler represents a change of one order of magnitude (or power of 10), our scale is actually a logarithmic scale. Therefore, if the coefficient of a number were equal to five, we would not place it halfway between the two powers of 10.
Here is a sample of log log graph paper:
http://www.csun.edu/science/ref/measurement/data/graphpaper/log_log_numbered.pdf

Estimating Logs
Here's a rough way to estimate logarithms. Let's consider the distance that is half-way between 10^0 and 10^1, which is $10^{1/2}$.

Recall that the 1/2 power is the square root. What is $\sqrt{10}$? It is a little more than $\sqrt{9}$, which equals 3.
So $10^{1/2}$ is close to 3. The actual logarithm of 3 is .48
Recall that the definition of a logarithm is the power to which 10 must be raised to equal that number.

The half-way number between 10^0 (or 1) and 10^1 (or 10) is $10^{1/2}$ (~3).

The half-way number between 10^1 (or 10) and 10^2 (or 100) is $10^{3/2}$ (~30).
The half-way number between 10^2 (or 100) and 10^3 (or 1000) is $10^{5/2}$ (~300).
On our scale, in which each cm constitutes an entire order of magnitude, we can use this half-way mark to quickly estimate our logs.
Here's an example:

Moon diameter
3.5×10^5 m

For our moon example above, the coefficient of our measurement is 3.5, and our base exponent is 5. Knowing that we can estimate the log of 3 to about 1/2, we can quickly plot this moon diameter half way between 10^5 and 10^6, at $10^{5.5}$.

Calculating Logs
Here is a way to calculate the logs of our measurements:

Let's review the Laws of Exponents:

$$x^a \bullet x^b = x^{a+b}$$

This statement is true when the bases are the same.

Now let's try to put one of our measurements into this form:

Moon diameter
3.5×10^5 m

We want to find a value for 3.5 that has a base of 10 raised to some power.

$$3.5 = 10^?$$

We know this value will be between zero and one, since

$1 < 3.5 < 10$, substituting $1 = 10^0$ and $10 = 10^1$

then $10^0 < 10^? < 10^1$

The power to which 10 must be raised is the logarithm of 3.5

$$3.5 = 10^{.54}$$

Now we'll use the laws of exponents:

$$\begin{aligned} & 3.5 \times 10^5 \\ = \quad & 10^{.54} \times 10^5 \\ = \quad & 10^{.54+(5)} \\ = \quad & 10^{5.54} \end{aligned}$$

Common Logarithms:

Log 1 = 0	Log 2 = .3	Log 3 = .48	Log 4 = .6
Log 5 = .7	Log 6 = .78	Log 7 = .85	Log 8 = .9
Log 9 = .95	Log 10 = 1		

And he is is

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Atomic nucleus	1 femto-m	0.000000000000001 m	1. 0x 10^{-15} m
Atom	100 pico-m	0.0000000001 m	1.0 x 10^{-10} m
Diameter of a C nanotube	1 nm	0.000000001 m	1.0x 10^{-9} m
Length of the sucrose molecule	1 nm	0.000000001 m	1.0x 10^{-9} m
Approx. width of hemoglobin molecule	10 nm	0.00000001 m	1.0x 10^{-8} m
HIV virus	100 nm	0.0000001 m	1.0x 10^{-7} m
Wavelengths visible light	400-700 nm	0.0000004 m	4.0x 10^{-7} m
		to 0.0000007 m	to 7.0 x 10^{-7} m
Bacteria	1 μm	0.000001 m	1.0 x 10^{-6} m
Fog droplet	10 μm	0.00001 m	1.0 x 10^{-5} m
Cell diameter	10 μm	0.00001 m	1.0 x 10^{-5} m
Dust particle	100 μm	0.0001 m	1.0 x 10^{-4} m
Diameter of a pinhead	1 mm	0.001 m	1.0 x 10^{-3} m
Width of fingernail	1 cm	0.01 m	1.0 x 10^{-2} m
Diameter of baseball	7.6 cm	0.076 m	7.6 x 10^{-2} m
Length of toad	10 cm	0.1 m	1.0 x 10^{-1} m
Meter stick	1 m	1.0 m	1.0 X 10^{0} m
Adult human	1.8 m	1.8 m	1.8 x 10^{0} m
Length of giant squid	10 m	10 m	1.0 X 10^{1} m
Soccer field length	100 m	100 m	1.0 X 10^{2} m
Height of tallest tree	100 m	100 m	1.0 X 10^{2} m
Length of George Washington Bridge, NYC	1 km	1000 m	1.0 X 10^{3} m
Height of Mount Everest	10 km	10,000 m	1.0 X 10^{4} m
Diameter of Island of Hawaii	100 km	100,000 m	1.0 X 10^{5} m
Diameter of large hurricane	1000 km	1,000,000 m	1.0 X 10^{6} m
Moon diameter	3476 km	347,600,000 m	3.476 X 10^{6} m
Earth diameter	12,756 km	12,756,000 m	1.2756 X 10^{7} m
Saturn diameter	120,536 km	120,536,000 m	1.206 x 10^{8} m
continued			

Earth – Moon distance	382,500 km	382,500,000 m	3.825×10^{8} m
Sun’s diameter	1,390,000 km	1,390,000,000 m	1.39×10^{9} m
Venus to the Sun distance	100 Giga-m	100,000,000,000 m	1.0×10^{11} m
Earth to Sun distance	150 Giga-m	150,000,000,000 m	1.5×10^{11} m
Solar system diameter	11.8 Tera-m	11,800,000,000,000 m	1.18×10^{13} m
1 light year	10 Peta-m	10,000,000,000,000,000 m	1.0×10^{16} m
Distance to nearest star	39.9 Peta-m	39,900,000,000,000,000 m	3.99×10^{16} m
Diameter of Milky Way	100,000 light years		
	100 Zeta-m	1,000,000,000,000,000,000,000 m	1.0×10^{21} m

What’s Going On?

In this activity, we have created a form of a slide rule. By using a logarithmic scale, we are able to compare over 40 orders of magnitude on a relatively short scale. Each centimeter on the scale represents one order of magnitude, or one power of 10. When we slide the right-hand ruler relative to the left-hand ruler, the ratios of adjacent measurements remain constant. For example, when we slide the right-hand ruler so that the Earth’s diameter on the left-hand side lines up with the diameter of a pinhead on the right-hand side, we are expressing a ratio comparing the sizes of these two objects. The diameter of the earth is approximately 10,000 m, or 10^{7} m, and the diameter of the pinhead is approximately 1 mm, or 10^{-3} m.

$$\frac{\text{Diameter of the earth}}{\text{Diameter of a pinhead}} \qquad \frac{10^{7}\text{ m}}{10^{-3}\text{ m}} = 10^{10}$$

The ratio of the sizes of these two objects is 10^{10}.
With the rulers in this position, this ratio, 10^{10}, will be constant between any two objects that line up with one another.

The objects on the list can be grouped into three different categories: the very small, the human scale, and the very large. This activity allows us to take the very small and the very large and bring them into the human scale, so that we can compare the relative sizes of things.

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